

Nonuniqueness of Kivelson, Kallin, Arovas and Schrieffer's fractional charge.

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It is found that the magnetic length has not been treated correctly to calculate the classical action. In fact, the charge and the magnetic length have not been resolved. It is of serious consequences, because fractional charge completely disappears and only the flux area, l_o^2 becomes fractional. The results of Kivelson et al are therefore not unique.

1. Introduction

Kivelson et al¹ suggest that for certain filling factors, $\nu_c = n/m$, the ratio of two integers, the energy is stable so that the Wigner crystal is unstable for $\nu \neq \nu_c$. We find that this result is obtained by ignoring l_o which is the magnetic length. When this magnetic length is considered properly, the need for the fractionally charged particles disappears. But there are fractionally charged quasiparticles, then what they are due to?

In this paper, we correct the paper of Kivelson et al. We introduce the magnetic length where it was left out but then the interpretation changes. Therefore, we are also able to correct the interpretation.

2. Theory

The path-integral representation of the partition function, $Z = Tr \exp(-\beta H_N)$ is obtained from the N particle hamiltonian,

$$H_N = \sum_{i=1}^N \frac{1}{2m^*} [p_i + \frac{e}{2c} \vec{B}_o \hat{z} \times \vec{r}_i]^2 + \sum_{j < k} V_2(\vec{r}_j - \vec{r}_k). \quad (1)$$

The lowest Landau level (LLL) is given by,

$$\phi_R(r) = (2\pi)^{-1/2} \exp[-\frac{1}{4}(\vec{r} - \vec{R})^2 + \frac{1}{2}i(\vec{r} \times \vec{R}) \times \hat{z}]. \quad (2)$$

Since Kivelson et al suggest $l_o = 1$, the above wave function is dimensionally not correct, so it may be assumed that when necessary the dimensions will be corrected. The path integral representation for Z is given by,

$$Z(\nu) = (1/N!) \Sigma_{P \in S_N} (-1)^P \int d^{2N} r < j | \exp[-\beta H_N] | P(i) > \quad (3)$$

where,

$$N = \nu B_o / \phi_o. \quad (4)$$

The inconsistencies in the formulas as well as in discussions are clearly visible. In (3), N is a pure number so that its factorial is well defined but in (4) N is actually a number per unit area, i.e., the number density but the factorial is not defined for the number density which need not be an integer. The action for a continuous path is defined as $\int L dt$ where

L is the Lagrangian of the system as,

$$S[R] = (1/2) \int_o^\beta d\tau [-i \Sigma_{j=1}^N (\dot{\mathbf{R}}_j \times \mathbf{R}_j) \cdot \hat{z} + \Sigma_{j \neq k} V(\mathbf{R}_j - \mathbf{R}_k)] \quad (5)$$

where V is the matrix element of the Coulomb potential between coherent states with τ as the imaginary time,

$$V(R) = (1/2) \sqrt{\pi} (e^2/\epsilon) \exp(-R^2/8) I_o(R^2/8). \quad (6)$$

Here the prefactor on the right hand side does not have the dimensions of a potential energy. The factor e^2/ϵ should be replaced by $e^2/(\epsilon l_o)$. This correction is very important because the correction can now occur in l_o otherwise only the charge can be corrected. Similarly, the argument of the exponential function requires to be corrected. The partition function Z is evaluated by finding all paths $R^c(\tau)$ for which the action has an extremum value. Here $R^c(\tau)$ is a vector function with $2N$ components $R_j(\tau)$. Again, the number of components is not $2N$ because N should be treated as density. The number of electrons in the ring is L . The real part of the action is $\alpha_o(\nu)L$ and the imaginary part determines the time dependence. Kivelson et al suggest that the relaxation corrections are small but there is no particular reason to think that such quantities should be compared at all. For a phase transition $\alpha(\nu) < \alpha_c$ some critical value. It has been suggested that this inequality is most likely satisfied by $\nu = 1/3, 1/5, \dots, 4/9$, but the fractions $1/3$ etc are not derived.

Suppose that there is a local dilation of the Wigner crystal by an amount δA , then the phase acquired is $\Delta\theta = 2\pi B_o \delta A / \phi_o$. Kivelson et al suggest that this means that the quasiparticle charge is,

$$Q^* = \pm \nu e, \quad (7)$$

because one can absorb $\pm \nu$ in $\phi_o = hc/e$. Here, $\delta A e / hc$ is the quantity which is occurring in the algebra so that $\pm \nu$ need not be absorbed in e to change it to $\pm \nu e$. Instead of absorbing $\pm \nu$ in e , we can absorb it in δA . The charge e then remains unchanged and δA changes to $\pm \nu \delta A$. Kivelson et al's result, $Q^* = \pm \nu e$, is then not necessarily correct.

Kivelson et al have estimated the energy for creation of a quasiparticle as,

$$E_{qp}(\nu) \sim 0.5\nu^2 e^2 / \epsilon_o. \quad (8)$$

This is not having the correct dimensions. To set it right, we can change it to $E_{qp}(\nu) \simeq 0.5\nu^2 e^2 / \epsilon_o l_o$. Then charge can be e and only l_o is changed to l_o / ν^2 . Therefore, the arguments used to discuss the creation energy are not satisfactory. Similarly, it is found that the arguments used to discuss the fractional charge in Peierls distortion and in the calculation of Berry's phase are not correct.

3. Conclusions

Kivelson et al have calculated the classical action from which they claim that the quasiparticles are fractionally charged. We have checked their calculation and find that the quasiparticles need not be fractionally charged and Kivelson's results are not unique. There may be a fractional area instead of the fractional charge.

Schrieffer et al have published several papers dealing with the fractional charge. It has been reported that in the case of a Peierls distortion, the charge may be fractional. A calculation of the Berry's phase has been published and in the present case, the classical action has been calculated. In all the cases, a fractional charge has been reported. Upon closer scrutiny of the algebraic derivations, it has been found that the calculation has not been performed correctly. We have found that the calculations have errors. Su and Schrieffer² have discussed the Peierls distortion and Arovas et al³ have calculated the Berry's phase. We have shown⁴ that both of these papers of Schrieffer are in error. Similarly, Laughlin's paper⁵ is also not correctly written. The error has been made in such a way that "exactness" is not affected⁶.

The correct theory of the quantum Hall effect is given in ref.7

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4. References

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